on solid surfaces because of the rapidity and ease with which the power and the cavity geometry can be controlled, and also the number of times it repeats itself.

## LITERATURE CITED

1. G. A. Gulii (ed.), Equipment and Technological Processes Using the Electrohydraulic Effect [in Russian], Mashinostroenie, Moscow (1977).
2. M. S. Plesset and R. B. Chapman, "Collapse of an initially spherical cavity in the neighborhood of a solid boundary," J. Fluid Mech. 47, Pt. 2 (1971).
3. A. Shima and G. Sato, "The collapse of a bubble attached to a solid wall," Ingenieur Archiv., 48, No. 2 (1979).
4. V. V. Voinov and O. V. Voinov, "Calculation of the parameters of a high-speed jet formed when a bubble collapses, ${ }^{n}$ Zh. Prikh. Mekh. Tekh. Fiz., No. 3 (1979).
5. O. V. Voinov and V. V. Voinov, "The collapse of a cavitation bubble near a wall," Dokl. Akad. Nauk SSSR, 227, No. 1 (1976).
6. V. A. Burtsev and V. V. Shamko, "Collapse of a spherical cavity induced by an underwater spark close to a solid wall," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1977).
7. C. L. Kling and F. G. Hammitt, "A photographic study of spark-induced cavitation bubble collapse," Trans. ASME. Ser. D., J. Basic Eng., 4, 130 (1972).
8. V. V. Shamko and A. I. Vovehenko, "Effect of boundary surfaces on the development of an underwater spark discharge," in: Hydromechanics [in Russian], No. 34, Naukova Dumka, Kiev (1976).
9. G. A. Grivnin, S. P. Zubrilov, V.A. Larin, and K. K. Shalnev, "Investigation of the collapse mechanism of the nonspheric cavitation processes in liquids with different properties, " in: Proc. 6th Conf. Fluid Mech. Vol. 1, Budapest (1979).
10. Yu. A. Grivnin, S. P. Zabrilov, and V. A. Larin, "Effect of the physical properties of a fluid on the pulsation and destruction of nonspherical cavitation recesses," Zh. Fiz. Khim., 54, No. 1 (1980).
11. A. I. Vovchenko, V. V. Kucherenko, and V. V. Shamko, "Features of the space-time evolution of vapor gas cavities generated by an underwater spark discharge," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1978).

## ENERGY TRANSFER TO A PLANE INCOMPRESSIBLE

PISTCN UNDER DETONATION LOADING

## S. A. Kinelovskii

UDC 534.222.2

Among the problems of explosion-produced acceleration, a special place is occupied by the problem of the one-dimensional projection of a flat plate or piston. One-dimensional problems are of interest because they are relatively simple to investigate theoretically. Moreover, one-dimensional projection is a method that lends itself to direct practical realization and constitutes a simplified model of many actual problems of explosive propulsion.

The analytic approach to the solution of one-dimensional problems is usually based on the following assumptions: The piston material is incompressible; the shock waves in the explosion products (EP) are weak; and the EP formally satisfy the equation of state of a perfect gas with adiabatic exponent $k=3$. The last two assumptions imply that the characteristics of the equations of motion of the gas are linear and do not change their slope on intersection with shock waves moving in the opposite direction (compression waves), and that throughout the process the pressure and speed of sound in the gas are related by the expression

$$
p=A c^{3}
$$

where the constant $A$ is determined by the initial thermodynamic state of the EP.
These assumptions have been used to obtain analytic solutions to a number of problems of the motion of a plane piston propelled by the explosion of a layer of explosive of finite thickness. The situation where a detonation wave impinges on the piston was considered in [1, 2]. A similar problem, with the difference that detona-

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 120-126, September-October, 1982. Original article submitted November 4, 1981.
tion is initiated at a rigid fixed wall, was examined in [3]. In [4] two plane pistons separated by the products of instantaneous detonation of an explosive charge were propelled in opposite directions as a special case the mass of one of the pistons was taken equal to zero). The case where detonation is initiated in the plane of the piston and propagates in the direction of the free end of the charge was studied in [5]. Moreover, in a number of investigations the process of acceleration of the piston was not actually considered, the limiting velocity of the piston being found directly from various physical considerations. All this work was reviewed in [6, 7].

We will consider the case where the end of the charge away from the piston is in contact with a vacuum. The three known methods of piston propulsion under these conditions were described and analyzed in [5]. Below, various other methods of propulsion of a plane piston are investigated under the assumptions previously specified. The problems are considered in dimensionless form: As the unit of length we shall take the thickness $l_{0}$ of the layer of explosive, as the unit of velocity the detonation velocity D , as the unit of mass the mass of the charge (per unit area); time is referred to the quantity $l_{0} / \mathrm{D}$, and pressure to $\rho_{0} \mathrm{D}_{2}$, where $\rho_{0}$ is the density of the explosive.

In [5] it was noted that in the various stages of acceleration of the piston its motion can be described by two types of solutions. This also holds good for the problems considered below, though the solutions are written in a canonical form somewhat different from that of [5].

In regions where the value of the invariant carried by the characteristics overtaking the piston is constant over the entire region the solution for the coordinate $X(t)$ and the velocity $U(t)$ of the piston can be written in the form

$$
\begin{equation*}
X(t)=\gamma\left[\tau+(2 / a)\left(S_{2}-\sqrt{a \tau+S_{1}}\right)\right], \quad U(t)=\gamma\left[1-1 / \sqrt{a \tau+S_{1}}\right] \tag{1}
\end{equation*}
$$

and in regions where the acceleration of the piston is effected in the rarefaction wave reflected from the piston the solution has the form

$$
\begin{equation*}
X(t)=\gamma\left[K_{2} \tau-\sqrt{(4 / a)\left(K_{1} \tau^{2}-\tau\right)}\right]-x_{0} \quad U(t)=\gamma\left[K_{2}-\left(2 K_{1} \tau-1\right) / \sqrt{a\left(K_{1} \tau^{2}-\tau\right)}\right] . \tag{2}
\end{equation*}
$$

The values of the parameters entering into these solutions are determined specifically for each problem. Solution (2) describes inter alia, the concluding phase of acceleration of the piston after the arrival of the rarefaction wave from the lefthand free end of the explosive. In this case it continues to hold good as $t \rightarrow \infty$ and can be supplemented by the expression for the limiting velocity of the piston

$$
\begin{equation*}
U_{\infty}=\gamma\left(K_{2}-2 \sqrt{K_{1} / a}\right) . \tag{3}
\end{equation*}
$$

Retaining the cnumeration of the problems adopted in [5], we will consider various, including the known, methods of propelling a plane piston of mass $M$.

1. Detonation Initiated at the Free End of the Charge

The charge is initiated at $t=0$ in the section $x=-1$. At $t=1$ the detonation wave reaches the piston $(x=0)$ and is reflected as a shock wave. For $t \geq 1$ the motion of the piston is described by solution (2) with

$$
\gamma=x_{0}=1, \quad a=32 /(27 M), \tau=t, \quad K_{1}=1+1 / a, \quad K_{2}=1+2 / a .
$$

The limiting velocity of the piston is given by expression (3).
2. Detonation Initiated in the Plane of the Piston

The charge is initiated at $t=0$ in the section $x=0$. At time $t=1$ the detonation wave reaches the free end of the charge $(x=-1)$ and generates a centered rarefaction wave whose leading front overtakes the already moving piston at $t=t_{1}$. In this problem $\gamma=1 / 2, a=8 /(27 \mathrm{M})$. At $0 \leq \mathrm{t} \leq \mathrm{t}_{1}=3+9 a / 4$ the motion of the piston is described by solution (1): $\tau=t, S_{1}=S_{2}=1$, and at $t \geq t_{1}$ by solution (2): $\tau=t-1, x_{0}=1, K_{1}=8\left(9 a^{2}+10 a+2\right) /\left[a\left(8+9 a^{2}\right], \mathrm{K}_{2}=1+\right.$ $4(2+3 a) /[a(8+9 a)]$ 。
3. Instantaneous Detonation

At time $t=0$ a layer of gas (instantaneous detonation products) of unit width is in contact with the lefthand face of the piston at $x=0$ 。 The piston is accelerated by the expansion of the compressed gas. At time $t=t_{1}$ the leading characteristic of the centered rarefaction wave from the free end reaches the piston and the subsequent acceleration of the piston takes place in the reflected rarefaction wave.

In this problem $\gamma=c_{0}=\sqrt{3 / 8}, a=1 /(\sqrt{6 M})$. At $0 \leq t \leq t_{1}$ the motion is described by solution (1):

$$
\tau=t, S_{1}=S_{\mathrm{a}}=1, t_{\mathbf{1}}=\left[1+a /\left(4 c_{0}\right)\right] / c_{0}
$$

and at $t \geq t_{1}$ by solution (2): $\tau=t, x_{0}=1$,

$$
K_{1}=8 c_{0}^{2}\left(a^{2}+4 a c_{0}+2 c_{0}^{2}\right) /\left[a\left(a+4 c_{0}\right)^{2}\right], \quad K_{2}=\left(a^{2}+8 a c_{0}+8 c_{0}^{2}\right) /\left[a\left(a+4 c_{0}\right)\right]
$$

From (3), with allowance for the values obtained for the constants, for the limiting velocity of the piston we have

$$
\begin{equation*}
U_{\infty}=\sqrt{3 / 8}\left(1+12 M+18 M^{2}-6 M \sqrt{2+12 M+9 M^{2}}\right) /(1+6 M) \tag{4}
\end{equation*}
$$

By means of simple algebra it is possible to show that for problems 1 and 2 the above solutions coincide with those presented in [5], and for problem 3 with the solution given in [8] and, moreover, with an earlier known solution of the problem [4].*
4. Charge Initiated Simultaneously at the Free End
and in the plane of the piston
At $t=0$ detonation is initiated in the charge from two ends at once: In the plane of the piston ( $x=0$ ) and at the free end $(x=-1)$. As in problem 1, the detonation wave from the free surface is associated with acentered rarefaction wave, generated by the expansion of the gaseous $E P$ into the vacuum. At $t=1 / 2$ the two detonation waves meet in the section $x=-1 / 2$. The meeting of the detonation waves generates two shock waves, one traveling towards the piston, the other towards the free end. At time $t=t_{1}$ the former overtakes the piston and is reflected from it, again as a shockwave, causing further acceleration of the piston.

In this problem $\gamma=1 / 2, a=8 /(27 \mathrm{M}), \mathrm{t}_{1}=2[2(1+a-\sqrt{17+8 a}) /(8-a)]^{2}$. The first stage $\left(0 \leq t \leq t_{1}\right)$ of piston acceleration is described by solution (1), in exactly the same way as in problem 2. At $t \geq t_{1}$ the motion of the piston is described by solution (2):

$$
\begin{gathered}
\tau=t, \quad x_{0}=1, \quad K_{1}=\frac{a}{b^{2}-1}\left\{1+b^{2}\left(b^{2}-1\right) /\left[2 a b-(b-1)^{2}\right]^{2}\right\}, \\
K_{2}=\frac{1}{b^{2}-1}\left\{2 a+(b-1)^{2}+2 b\left(b^{2}-1\right) /\left[2 a b-(b-1)^{2}\right]\right\}, b=\sqrt{a t_{1}+1} .
\end{gathered}
$$

In Fig. 1 we have plotted the curves, constructed from (1), of $U_{\infty}$ versus the relative mass of the piston $M$ for the various problems there and in what follows the number of the curve in the figures corresponds to the number of the problem). It is clear from Fig. 1 that the $U_{\infty}(M)$ curve for problem 4 lies more or less half way between the curves for problems 1 and 2. From the solution of problem 4 it follows that the two-stage acceleration obtained in this case ensures projection velocities similar to those achieved in problem 1, where the acceleration conditions are most "severe" [5]. However, the distance over which the piston is accelerated to a velocity equal to $(0.7 \sim 0.9) \mathrm{U}_{\infty}$ is two to four times greater than in problem 1. In Fig. 2, for two values of M and various problems, we have reproduced the calculated dependence of piston velocity on path traveled, from which it is possible to judge the nature of the acceleration (continuous curves: $M=0.1$; dashed curves $\mathrm{M}=0.6$ ).

## 5. Detonation Initiated within the Charge

At the initial instant $t=0$ detonation is initiated in a section at a distance $l$ from the piston and detonation waves depart in both directions. In this stage the process is symmetrical about the section $x=-l$, i.e., is equivalent in each direction to the propagation of a detonation wave from a fixed rigid wall. In this case (see, for example, [3]) there extends from the wall a region of rest occupying at a given moment of time half the distance to the detonation front. At time $t=l$ the righthand detonation wave reaches the piston $(x=0)$, being reflected from it as a shock wave and initiating the piston acceleration process. At time $t_{0}=1-l$ the lefthand detonation wave reaches the free end of the charge $(x=-1)$ and is reflected from it as a centered rarefaction wave whose leading front may, in principle, overtake the piston. In this problem $\gamma=1 / 2, a=8(27 \mathrm{M})$. The piston acceleration process depends largely on whether or not the piston is overtaken by the leading characteristic of the "region of rest ${ }^{n}$ (as in [3]). In the case of very light pistons, when $a \geq 2 / l$, i.e., $M \leq 4 l / 27$, this characteristic does not overtake the piston (nor does any other perturbation). There is then only one piston acceleration stage which for any $t \geq l$ is described by solution (2):

$$
\tau=t, x_{0}=l_{x} K_{1}=(1+4 a l) /\left(4 a l^{2}\right), K_{2}=(1+2 a l) /(a l)
$$

* We note that in [4] there is a misprint in the expression for $\mathrm{U}_{\infty}$, corresponding to (4): the coefficient 16 in the radicand should read 12 .


Fig. 1


Fig. 2


Fig. 3
The limiting velocity of the piston is given by expression (3).
In the case of a heavier piston $(a<2 / l)$ the region of rest and then the centered rarefaction wave overtake the piston at times $t_{1}$ and $t_{2}$, respectively, and the piston acceleration process is divided into several stages. At $l \leq t \leq t_{1}=4 l(2-a l)$ the motion of the piston is described by the above solution for light pistons. At $t_{1} \leq t \leq t_{2}=$ $\left(a^{2} b^{2}-\mathrm{S}_{1}\right) / a$, where $\mathrm{b}=1+\mathrm{S}_{2} / a+\mathrm{t}_{0} / 2$, the motion of the piston is described by solution (1):

$$
\tau=t, S_{1}=4\left(1+2 a^{2} l^{2}\right) /(2-a l)^{2}, S_{2}=\left(2+a^{2} l^{2}\right) /(2-a l)
$$

And, finally, at $t=t \geq t_{2}$ the motion of the piston is again described by solution (2), where

$$
\tau=t-t_{0,} x_{0}=1, K_{1}=\left(\tau_{2}+a b^{2}\right) / \tau_{2}^{2}, K_{2}=1+2 a / \tau_{2}, \quad \tau_{2}=t_{2}-t_{0}
$$

These expressions for $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ and equation (3) determine the limiting piston velocity.
The brief existence in the gas of a "fixed wall," noted above in connection with problem 5, suggested that for $\mathrm{M} \geqslant 0$ the limiting piston velocity might be higher than in problem 1. However, an analysis of the solution showed that this is not so. On average, the effective gas pressure accelerating the piston is always less in problem 5 than in problem 1. This is illustrated, in particular, by Fig. 3, where the distribution of the speed of sound in the gas is shown for both problems: (a) at the time the detonation wave reaches the piston; (b) when a piston with $\mathrm{M}=0.1$ has traveled a path equal to 0.2 (for problem $5 l=0.5$ ). For other values of $l$ and M the qualitative picture is roughly the same.

From the solution of problem 5 it follows that this problem is intermediate with respect to both limiting velocity and piston acceleration regime between problems 1 and 2. In fact, varying the value of $l$ gives a set of piston acceleration regimes ranging from "mild ${ }^{n}\left(l=0\right.$, problem 2 ) to ${ }^{\text {"severe }}{ }^{n}(l=1$, problem 1$)$.

The analysis of problems 2 and 3 in [5] can be supplemented by a remark that also applies to problem 5. In these problems, by the time the rarefaction wave arrives from the free end of the charge the piston has been able to accumulate $95 \%(\mathrm{M}=0.1)$ to $80 \%(\mathrm{M}=1.4)$ of its limiting velocity. Thus on the range $\mathrm{M}<1.4$ the motion of the piston is mainly determined by the initial EP pressure pulse, the contribution of the residual pressure being fairly small.

## 6. Instantaneous Detonation across a Gap

At $t=0$ the interval $-1 \leq x \leq 0$ is occupied by compressed gas, the product of the instantaneous detonation of the explosive charge. The left-hand edge of the gas is free, and at the right-hand edge there is, in the general
case, an intermediate piston of mass $\mu$ (special case: $\mu=0$ ). In the section with coordinate $\mathrm{x}=\delta_{0}$ we have the main piston with mass ( $M-\mu$ ). Both pistons are incompressible, and between them and to the left of the gas there is a vacuum.

At $t \geq 0$ the motion of the intermediate piston is described by the solution of problem 3 with $\alpha=\alpha=1 /$ $(\sqrt{6 \mu})$. At time $t_{0}$ the intermediate piston reaches the main one. The impact is assumed to be absolutely inelastic, and from this condition we find the initial velocity $U_{0}$ of a main piston of mass $M$; when the pistons meet a shock wave enters the gas. The mass of the intermediate piston $\mu$ is assumed to be small enough for the unloading wave from the free edge not to overtake it before impact. At $t=t_{1}$ the piston overtakes the leading front of the centered rarefaction wave from the free edge.

In this problem

$$
\gamma=c_{0}=\sqrt{3 / 8}, a=1 /(\sqrt{6} M) .
$$

From its solution we have

$$
t_{0}=\delta_{0}(1+2 \lambda) / c_{0}, U_{0}=a c_{0} /[\alpha(1+\lambda)], \lambda=\sqrt{c_{0}\left(\alpha \delta_{0}\right)} .
$$

At $t_{0} \leq t \leq t_{1}$ the motion of the piston is described by solution (1), where

$$
\begin{gathered}
t_{1}=t_{0}+\frac{1}{a}\left[\left(\frac{a\left(1-2 \lambda \delta_{0}\right)}{2 c_{0}}+\sqrt{S_{1}}\right)^{2}-S_{1}\right], \quad \tau=t-t_{0} \\
S_{1}=\left[c_{0} /\left(c_{0}-U_{0}\right)\right]^{2}, S_{2}=a \delta_{0}^{\prime}\left(2 c_{0}\right)+\sqrt{S_{1}}
\end{gathered}
$$

and at $t \geq t_{1}$ by solution (2), where

$$
K_{1}=\left(2 t_{1}-t_{0}+S_{1} / a\right) / t_{1}^{2}, \quad K_{2}=1+\left(1 / c_{0}+2 S_{2} / a-t_{0}\right) / t_{1}
$$

7. Detonation across a Gap: Detonation Initiated
at End of Charge away from Piston
This problem is similar in formulation to the previous one, differing only with respect to the method of initiation of the charge. The motion of the intermediate piston of mass $\mu$ begins with the arrival of the detonation wave $(t=1)$ and is described by the solution of problem 1. At time $t_{1}$, after absolutely inelastic impact, the main piston of total mass $M$ begins to move with initial velocity $\mathrm{U}_{0}$. As in problem 1 , there is only one acceler~ ation stage, which at $t \geq t_{1}$ is described by solution (2):

$$
\begin{gathered}
a=32 /(27 M), \alpha=32 /(27 \mu), \tau=t, \gamma=x_{0}=1, \\
t_{1}=1+\delta_{0}+2 \delta_{0}\left[1+\sqrt{1+\alpha\left(1+\delta_{0}\right) / \delta_{0}}\right] / \alpha, \\
V_{0}=\frac{a}{\alpha}\left[1-\frac{2\left(t_{1}+\delta_{0}\right)}{(\alpha+2) t_{1}-\alpha\left(1+\delta_{0}\right)}\right], \\
K_{1}=\frac{1}{t_{1}}\left[1+\frac{t_{1}}{a\left(1+\delta_{0}-U_{0} t_{1}\right)^{2}}\right], \quad K_{2}=\frac{1}{t_{1}}\left[1+\delta_{0}+\frac{2\left(t_{1}+\delta_{0}\right)}{a\left(1+\delta_{0}-U_{0} t_{1}\right)}\right] .
\end{gathered}
$$

The formulation of the last two problems is based on the physical fact that introducing a gap increases the amount of gas whose mass velocity is directed towards the piston and hence may increase the momentum imparted to the piston by the gas. This principle is confirmed by the solutions obtained, which for both problems give qualitatively similar results.

In the absence of an intermediate piston $(\mu=0)$ introducing a gap gives an increase in the final piston velocity that grows with the gap. For problem 6 this is illustrated by Fig. 4 which shows $U_{\infty}$ (M) at various $\delta_{0}$ for the case $\mu=0: \delta_{0}=0$ (problem 3) - curve $3, \delta_{0}=1$ and $\delta_{0}=2$ - curves 6 a and 6 b , respectively. The corresponding results for problem 7 are presented in Fig. 1: curve $7 \mathrm{a}-\delta_{0}=0.5$, curve $7 \mathrm{~b}-\delta_{0}=2$. Figure 5 gives the $\mathrm{U}_{\infty}\left(\delta_{0}\right)$ curves for various cases. Here, the continuous curves relate to problem 7 and the dashed curves to problem 6; for the upper group of curves (continuous and dashed) $\mathrm{M}=0.1$, for the lower group $\mathrm{M}=1.0$; the mass of the intermediate piston $\mu=0$ (curves a), 0.1 (curves b) and 0.25 (curves c).

The problems considered show that there are fairly broad possibilities of controlling the projection process. Initiating detonation within the charge (problem 5) makes it possible to achieve almost any (from "mild" (problem 2 to "severe" (problem 1)) piston acceleration conditions. Initiating the charge from both ends (problem 4) gives two-stage piston acceleration and ensures a final piston velocity closer to the case represented by problem 1 under substantially milder acceleration conditions. Conversely, introducing a vacuum gap between piston and charge makes it possible, in some cases (problem 7), to obtain more severe acceleration conditions


Fig. 4


Fig. 5
and achieve higher piston velocities than in problem 1 or, for about the same velocity, markedly increase the path on which this velocity is reached.

It should be noted that, in actual fact, obtaining higher projection velocities with a gap between piston and charge is, generally speaking, problematic. Investigations [9, 10] of the projection of plates across an air gap did not demonstrate an increase in the final velocity of the plates, and in [9], where the mass of the plate was fairly large ( $M \approx 1.35$ ), the final velocity decreased with increase in the gap. A comparison of the results of these studies and the solutions of the model problems examined above leaves some uncertainty as to the importance of the presence or absence of air in the gap. Experiments [9] showed that there is a marked increase in plate velocity when the air is evacuated from the gap. If it is assumed that the presence of air in the gap is to some extent equivalent to the presence of an intermediate piston, then, even when the mass of this piston is very small, it follows from the solutions of the problems that the velocity of the main piston remains approximately the same as in the absence of a gap, or may even fall somewhat if the gap is small, and begins to increase only at sufficiently large gaps, when the one-dimensional process assumed in the problems is rather difficult to achieve experimentally. With increase in the gap the acceleration time increases, which corresponds qualitatively to the results of $[9,10]$. Thus, in this case the experimental results are in qualitative agreement with the solution of the model problems, if in the latter an attempt is made to take account of the air in the gap. On the other hand, if we consider the experimental results of [10], for equally thick steel plates, then from the solution of problem 7 with $\mu=0$ we find that the calculated ratio of piston velocities with and without a $10-\mathrm{mm}$ gap ( $\delta_{0}=0.25$ ) is equal to 1.05 (i.e., the calculated velocity increment is in fact small), the calculated value of the absolute piston velocity being close to the $4.1 \mathrm{~km} / \mathrm{sec}$ recorded in the experiments. Here a certain agreement between the results of the experiments and the solution of the problem is achieved without taking into account the presence of air in the gap. Thus, so far the comparison does not suggest any serious qualitative contradiction between the solutions of model problems 6 and 7 and the known experimental results.

The author is grateful to Yu . A. Trishin for his interest in the work and useful discussions.

## LITERATURE CITED

1. $\mathrm{K}_{\mathrm{s}}$ P. Stanyukovich, Nonsteady Motions of a Continuum [in Russian], Gostekhizdat, Moscow (1955).
2. A. Aziz, H. Hurwitz, and H. Sternberg, "Energy transfer to a rigid piston under detonation loading, ${ }^{\text {a }}$ Phys. Fluids, 4, No. 3 (1961).
3. S. Abarbanel and G. Zwas, "The motion of shock waves and products of detonation confined between a wall and a rigid piston, ${ }^{n}$ J. Math. Anal. Applicat., 28, No. 3 (1969).
4. G. M. Lyakhov, "Directed projection of bodies by explosion products, ${ }^{2}$ Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1962).
5. A. G. Ivanov and G. Ya. Karpenko, "Acceleration of thin plates by explosion products using various methods of charge initiation, ${ }^{17}$ Fiz. Goreniya Vzryva, 16, No. 2 (1980).
6. A. A. Deribas, Physics of Explosion Hardening and Explosion Welding [in Russian], Nauka, Novosibirsk (1980),
7. G. E. Kuz $\mathrm{Imin}^{\text {min }}$ Use of numerical methods in problems of explosive pressing and welding, Candidate ${ }^{p}{ }^{p}$ Dissertation Physicomathematical Sciences, Institute of Hydrodynamics, Siberian Branch, Academy of Sclences of the USSR, Novosibirsk (1978).
8. Corrigendum to the article by A. G. Ivanov and G. Ya. Karpenko, Fiz. Goreniya Vzryva, 17, No. 1 (1981).
9. V. I. Tsypkin, V. N. Mineev, et al., "Investigation of the acceleration of copper plates by explosion products across a gap," Zh. Tekh. Fiz., 45, No, 3 (1975).
10. V.A. Ogorodnikov, S. Yu. Pinchuk, et al., "Experimental-theoretical investigation of the acceleration of plates by explosion products across a gap," Fiz. Goreniya Vzryva, 17, No. 1 (1981).

DETERMINATION OF THE SPALL STRENGTH
FROM MEASURED VALUES OF THE SPECIMEN
FREE-SURFACE VELOCITY
S. A. Novikov and A. V. Chernov

U DC 539.412:539.42

Measurements of the free-surface velocity on reflection of a nonstationary shock wave make it possible to obtain the data needed to determine the spall strength of a material $\sigma_{0}$, which is calculated from the expressions [1]

$$
\begin{align*}
& \sigma_{0}=\rho_{0} C_{0}\left(W_{0}-W_{k}\right) / 2 ;  \tag{1}\\
& \sigma_{0}=\rho_{0} C_{0}\left(W_{0}-\bar{W}\right), \tag{2}
\end{align*}
$$

where $\rho_{0}$ is the initial density of the material; $\mathrm{C}_{0}$, velocity of the plastic waves in $i t ; W_{0}$, maximum of the specimen free-surface velocity realized on arrival of the shock wave at that surface; $W_{k}$, value at the first minimum of the free-surface velocity time dependence; $\bar{W}$, mean velocity of the spall fragment. The values of $W_{0}$ and $W_{k}$ are determined from the continuously recorded free-surface velocity measured by the capacitive transducer method [2]; $W_{0}$ can also be determined as the velocity of a thin artificial (prepared) spall fragment, i.e., a thin foil of the same material fitted tightly to the specimen; $\bar{W}$ is the usual mean spall velocity.

The literature does not contain any analysis of the limits of applicability of these expressions or the assumptions made in deriving them. We have the refore investigated the nature of the underlying assumptions and the limits of applicability of the equations derived.

Using the method of characteristics [3], let us consider the flow in a specimen subjected to spalling in the plane wave formulation. The $X-T$ flow diagram is reproduced in Fig. 1a, where $X$ is the Euler coordinate and $T$ is time. We assume that the material fails instantaneously in a certain plane the point $F$ on the $X-T$ diagram), as soon as the tensile stress in that plane reaches the value $\sigma_{0}$. This condition is first realized on the last C -characteristic OF of the centered rarefaction wave LOF formed when the shock wave SO reaches the free surface. The spall shock propagates from the fracture point $F$ within the spall plate (to the right).


Fig. 1

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 126-129, September-October, 1982. Original article submitted June 23, 1981.

